Maths Notes: Calculus First Principles

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$\cos x$

Let:
- $f(x) = \cos x$
- $f(x + h) = \cos x + h$

Use Formula:
- $\lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \frac{\cos x + h - \cos x}{h}$
- $= \frac{-2\sin\left(\frac{a+b}{2}\right)\sin\left(\frac{a-b}{2}\right)}{h} = -2\sin\frac{2x+h}{2} \cdot \frac{\sin\frac{h}{2}}{\frac{1}{2}h} \cdot \frac{1}{2}$

Limit: h ---> 0
- $= -2\sin\frac{2x+0}{2} \cdot 1 \cdot \frac{1}{2}$
- $= -2\sin x \cdot 1 \cdot \frac{1}{2}$
- $= -\sin x$

Therefore: $\frac{dy}{dx} = -\sin x$

$\sin x$

Let:
- $f(x) = \sin x$
- $f(x + h) = \sin x + h$

Use Formula:
- $\lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \frac{\sin x + h - \sin x}{h}$
- $= \frac{2\cos\left(\frac{a+b}{2}\right)\sin\left(\frac{a-b}{2}\right)}{h} = 2\cos\frac{2x+h}{2} \cdot \frac{\sin\frac{h}{2}}{\frac{1}{2}h} \cdot \frac{1}{2}$

Limit: h ---> 0
- $= 2\cos\frac{2x+0}{2} \cdot 1 \cdot \frac{1}{2}$
- $= 2\cos x \cdot 1 \cdot \frac{1}{2}$
- $= \cos x$

Therefore: $\frac{dy}{dx} = \cos x$

$\frac{1}{x}$

Let:
- $f(x) = \frac{1}{x}$
- $f(x + h) = \frac{1}{x+h}$

Use Formula:
- $\lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \frac{\frac{1}{x+h} - \frac{1}{x}}{h}$
- $= \frac{x - x + h}{(x+h)(x)}$
- $= \frac{\frac{-h}{(x+h)(x)}}{h}$
- $= \frac{-h}{(x+h)(x)} \times \frac{1}{h}$

Limit: h ---> 0
- $= \frac{-1}{(x+0)(x)}$
- $= \frac{-1}{x^2}$

Therefore: $\frac{dy}{dx} = \frac{-1}{x^2}$

x^2

Let:
- $f(x) = x^2$ And
- $f(x + h) = (x + h)^2$
- $f(x + h) = x^2 + 2xh + h^2$

Use Formula:
- $\lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \frac{x^2 + 2xh + h^2 - x^2}{h}$
- $= \frac{2xh + h^2}{h}$
- $= 2x + h$

Limit: h ---> 0
- $= 2x + 0$
- $= 2x$

Therefore: $\frac{dy}{dx} = 2x$

x^3

Let:
- $f(x) = x^3$
- $f(x + h) = (x + h)^3$
- $f(x + h) = x^3 + 3x^2h + 3xh^2 + h^3$

Use Formula:
- $\lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \frac{x^3 + 3x^2h + 3xh^2 + h^3 - x^3}{h}$
- $= \frac{3x^2h + 3xh^2 + h^3}{h}$
- $= 3x^2 + 3xh + h^2$

Limit: h ---> 0
- $= 3x^2 + 3x(0) + (0)^2$
- $= 3x^2$

Therefore: $\frac{dy}{dx} = 3x^2$

$\sqrt{x}$

Let:
- $f(x) = \sqrt{x}$
- $f(x + h) = \sqrt{x + h}$

Use Formula:
- $\lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \frac{\sqrt{x+h} - \sqrt{x}}{h} \times \frac{\sqrt{x+h} + \sqrt{x}}{\sqrt{x+h} + \sqrt{x}}$
- $= \frac{x + h - \sqrt{x+h}\sqrt{x} + \sqrt{x+h}\sqrt{x} - x}{h\sqrt{x+h} + h\sqrt{x}}$
- $= \frac{h}{h\sqrt{x+h} + h\sqrt{x}}$

Limit: h ---> 0
- $= \frac{1}{\sqrt{x+0} + \sqrt{x}}$
- $= \frac{1}{2\sqrt{x}}$

Therefore: $\frac{dy}{dx} = \frac{1}{2\sqrt{x}}$